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Are Cosmic Strings Frustrated?

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Abstract

In models involving more than one $U(1)'$-charged scalar field [as is often the case in Grand Unified Theories] the cosmic strings which emerge are in general more complicated than simple Nielsen-Olesen flux tubes. In particular, strings often form with domain walls or with log-infinite contributions to the mass. A Universe with these strings is very different from one in which there are only Nielsen-Olesen-type flux tubes.

I. Introduction

In a schematic scenario for cosmic strings [1], one considers a single scalar field associated with the $U(1)'$ group that will become broken. The cosmic string is either a global vortex (if $U(1)'$ is an ungauged symmetry) or a Nielsen-Olesen flux tube [2] (if $U(1)'$ is gauged). In this letter, we consider models in which there are *many scalar fields with different charges* under $U(1)'$. This situation naturally arises in realistic models based on Grand Unified Theories (GUTs). When $U(1)'$ is gauged, the groundstate cosmic string is a Nielsen-Olesen flux tube. This groundstate configuration requires special relationships among winding numbers of the various fields. However, winding numbers are determined by random initial conditions for the fields at the time of the phase transition and are therefore statistical in nature. We argue that the conditions for a groundstate Nielsen-Olesen string are not, in general, satisfied at the time of string formation and that the types of structures which arise will be more complicated than the Nielsen-Olesen flux tube. Specifically, strings may form with associated domain walls or there may be log-infinite contributions to their mass as in the case of the global string. We refer to these possibilities as *frustration*.

In the simplest scenario for cosmic string formation, the $U(1)'$ symmetry is broken by a single scalar field Φ, when the temperature in the Universe drops below the critical temperature for the phase transition, T_c. Symmetry breaking is signaled when Φ acquires a vacuum expectation value (VEV), $<\Phi> = \mu e^{i\alpha}$. α is an arbitrary phase and will in general be uncorrelated over distances greater than the correlation length for the phase transition, ξ. The scale for ξ is set by either the critical temperature ($\xi \sim T_c^{-1}$) or the horizon size at that time, ($\xi \sim H(T_c)^{-1} \simeq t$), depending on the nature of the phase transition. As the field continues to evolve, much of the initial random variations in the field die out. However, topologically stable defects remain. In particular, if in going around a given loop in space, α changes by $2\pi N$, then passing through the loop will be some number of cosmic strings the sum of whose winding numbers is N. This picture for cosmic string formation was first discussed by Kibble [1,3].

Though, presumably, the formation of cosmic strings does not depend on $U(1)'$ being a gauged or global symmetry, the energetics of the strings in the two cases is markedly different. If $U(1)'$ is a global symmetry, then $\partial_\theta\Phi \to iN\mu/r$ for $r \to \infty$ where r is the distance from a given string and $\partial_\theta\Phi \equiv \frac{1}{r}\frac{\partial\Phi}{\partial\theta}$. The gradient energy, $\int d^2r\frac{1}{2}|\partial_\theta\Phi|^2$ leads to a contribution $\sim N^2\mu^2 ln(R/r_o)$ to the mass per unit length of the string. Here, R is an astrophysical scale (either the typical size of a loop or the mean separation between strings) and r_o is the core radius of the string. On the other hand, if $U(1)'$ is gauged, then a 'magnetic' flux tube of the gauge field A'_μ will be set up with $A'_\theta \to N/qr$ for $r \to \infty$. In this case, $D_\theta\Phi = (\partial_\theta - iqA'_\theta)\Phi$ falls off exponentially away from the core and there are no logarithmic contributions to the mass.

The simple situation described above is drastically altered when there are more than one $U(1)'$-charged scalar fields which acquire a VEV. Two important issues arise: (1) a ground state configuration consisting of a Nielsen-Olesen flux tube does not generally exist for arbitrary choices of winding numbers for the various fields and (2) the phases of the different fields may not be completely well correlated at the time of string formation. Field configurations are determined, at least initially, by statistical considerations. For example, the heaviest component fields will condense first as the Universe cools and form strings with somewhat random winding numbers. As light fields condense, the strings may become frustrated: given arbitrary winding numbers for the heavy fields, a true groundstate for the light fields may not exist without domain walls (or at least logarithmic contributions to the mass). Moreover, given a configuration for the heavy fields, the light fields may not be well correlated with the heavy fields. Therefore, even if a groundstate or Nielsen-Olesen string configuration exists, this groundstate may not be realized, at least for the initial string configuration. The phases of the different fields can be correlated directly through *local* scalar interaction terms. If $U(1)'$ is gauged, then the phases are also correlated indirectly through the gauge field. Since gauge interactions enter through the covariant derivative, they are *non-local* and are therefore efficient in setting up correlations among the various fields only within a region of size ξ, i.e., a correlation region. When phase correlations are either absent of inefficient, the winding numbers of the various fields will be independent and the strings will be, in general, frustrated.

A Universe with frustrated cosmic strings is very different from one with only Nielsen-Olesen strings. For example, domain walls are in general disastrous for cosmology [4]. Furthermore, while strings without domain walls are candidates for the seed fluctuations necessary to initiate galaxy formation, strings with attached domain walls do not have this appeal and are at best cosmologically 'safe' though in some cases they can be ruled out by observations [5]. Furthermore, frustration may spoil fermionic superconductivity in cosmic strings [6,7].

The outline of the paper is as follows: In section II we give simple examples of frustrated strings where, for the most part, we consider models with two scalar fields and one $U(1)'$ symmetry. In section III we discuss these examples in the context of standard GUTs based on the groups $SO(10)$ and E_6. Our ideas are summarized in section IV.

II. Simple Examples

Consider two complex scalar fields Φ and σ with $U(1)'$ charges q_Φ and q_σ respectively. Unless otherwise noted, we will assume that $U(1)'$ is a local (gauged) symmetry and that Φ and σ are neutral with respect to all other gauge groups. We write $\Phi = fe^{i\alpha}$ and $\sigma = ge^{i\beta}$. Furthermore, we take the scalar potential to be of the following form:

$$V(\Phi,\ \sigma) \;=\; V_{phase} \;+\; V_{mag} \tag{2.1}$$

where the magnitudes of Φ and σ in vacuum are determined by V_{mag} and correlations between the phases of Φ and σ are determined by V_{phase} . More precisely, we assume that V_{mag} is of the form

$$V_{mag} = \lambda(|\Phi|^2 - \mu^2)^2 + \delta(|\sigma|^2 - \eta^2)^2 \tag{2.2}$$

so that in vacuum, $f = \mu$ and $g = \eta$ [8]. V_{phase} contains $U(1)'$-invariant terms that are not of the form AA^* where A is some product of the fields. For example, if $q_\Phi = -q_\sigma$ then V_{phase} will be

$$V_{phase}(\Phi,\ \sigma) \;=\; \left(A_1|\Phi|^2 \;+\; A_2|\sigma|^2\right)\frac{\Phi\sigma}{2} \;+\; \frac{B}{2}(\Phi\sigma)^2 \;+\; \ldots \quad +\; h.c. \quad . \tag{2.3}$$

(We have implicitly assumed that the mass matrix has been diagonalized in Φ and σ.) Of course V_{phase} should be included in a proper calculation of the magnitudes of Φ and σ. For simplicity, we assume that $|V_{phase}| \ll |V_{mag}|$ so that V_{phase} need not be considered in determining the magnitudes of Φ and σ. The more general case is straightforward but tedious to analyze. Eqn(2.3) can then be written:

$$V_{phase}(\alpha,\beta) = a\cos(\gamma) \ + \ b\cos(2\gamma) \ + \ \dots \tag{2.4}$$

where $a = A_1\mu^2 \ + \ A_2\eta^2$, $b = B\mu^2\eta^2$, and $\gamma = \alpha + \beta$.

Consider the case where Φ is a 'heavy' field and σ is a 'light' field: $\mu \gg \eta$. Φ acquires its VEV early on, breaking $U(1)'$ and Φ strings form via the usual Kibble mechanism [3]. For these strings, $A'_\theta = m/r$ where $m = N_\Phi/q_\Phi$. [By N_X we mean the winding number of the X field.] Sometime later, σ acquires a VEV. As far as σ is concerned, Φ and A'_μ are fixed background fields. The question we now address is whether the strings in this model will be Nielsen-Olesen strings, i.e., whether the relation

$$\frac{N_\sigma}{q_\sigma} \ = \ m \ = \ \frac{N_\Phi}{q_\Phi} \tag{2.5}$$

is satisfied. A number of issues arise in attempting to answer this question and we will discuss these issues in the context of specific examples.

Let $q_\Phi \ = \ -q_\sigma \ \equiv \ e$ and consider the time when the Φ-strings have already formed and σ is acquiring its VEV. σ interacts with the Φ-string through couplings with Φ and A'_μ. First, let us assume that $V_{phase} \ = \ 0$ so that σ interacts with a Φ-string only through the gauge field. [Though the terms in V_{phase} are normally present, they might be forbidden by symmetries.] In the presence of a Φ string with winding number N_Φ, σ can minimize its gradient energy by choosing $N_\sigma \ = \ -N_\Phi$ (i.e., forming a groundstate (Nielsen-Olesen) string). A number of authors have used these energetic considerations to argue that σ necessarily chooses $N_\sigma \ = \ -N_\Phi$[6,7]. However, N_σ is determined by the orientation of σ far from the Φ-string. The gauge terms are non-local and act efficiently only up to a distance equal to ξ_σ, the correlation length for σ at the time when it acquires a VEV. If ξ_σ is much less than the average separation between Φ-strings ($\equiv L_\Phi$) then gauge interactions will not be able to force σ to wind. The winding number of σ, as determined by its orientation far from the Φ-string, will come from random initial conditions for the σ field and will not, in general satisfy $N_\sigma \ = \ -N_\Phi$. Of course, σ-strings will also form. The point is that the positions of the Φ and σ strings will be independent. The $N_\Phi \neq -N_\sigma$ string is one example of what we call cosmic string frustration.

The energetics of the Φ and σ strings are similar to the energetics of global strings. Keeping in mind that A'_μ is determined by the heavy Φ-field, we see that $|D_\theta\sigma| \rightarrow |N_\Phi + N_\sigma|\eta/r$ far from the core of a given string. Unless $N_\sigma = -N_\Phi$, there will be a contribution to the mass per unit length of the strings of order $(N_\Phi + N_\sigma)^2\eta^2 ln(R/r_o)$. $ln(R/r_o)$ can be $O(100)$ and this contribution should be compared with the other contributions to the mass which, for the Φ-strings, are of order μ^2. Perhaps more importantly, there will be long range forces acting between strings and these forces may affect the evolution of the string network. Moreover, as will be discussed below, the fact that σ may not wind in the presence of a Φ-string (and would therefore be non-zero in the core of the Φ-string) will have important implications if the Φ-string is to be a fermionic superconducting cosmic string [9].

Are the frustrated strings of the previous discussion stable? Consider, for example, a (1, 0) string. [We will label a string by the ordered pair (N_Φ, N_σ).] The mass per unit length of the string will be $\sim \mu^2 + \eta^2 \ln(R/r_o)$ where $r_o \sim 1/e\mu$ is the core radius of the magnetic flux tube for the string. Alternatively, consider a (1, -1) string and a (0, 1) string. The winding numbers for Φ and σ as determined on a loop enclosing the two strings are the same as those for the single (1, 0) string. The total mass per unit length for the two strings is $\sim \mu^2 + \eta^2 + \eta^2 \ln(R/r'_o)$ where $r'_o \sim 1/\delta^{1/2}\eta$ is the core radius for the σ-string. If $r'_o \gg r_o$ (*i.e.*, $\delta\eta^2 \ll e^2\mu^2$), then the total energy of the two strings in the second configuration will be less than that of the single (1, 0) string implying that the (1, 0) string is unstable and should 'decay' into the two string configuration. However, the energy difference between the two configurations is small ($\Delta E/E \sim \eta^2 \ln(r_o/r'_o)/\mu^2$) and the decay process is very complicated. In particular, this process involves the formation of a σ-string ($N_\sigma = 1$) and an antistring ($N_\sigma = -1$) with the antistring being laid on top of the Φ-string. Such a process requires that $|\sigma|$ go to zero where these strings are to form and this is very costly in terms of *local* energy. On the other hand, the difference between the energies of the two configuration comes from *non-local* energy considerations. The indication is that the (1, 0) string will be very long lived though a full understanding of this problem requires a more detailed analysis.

Now let us suppose that $\xi_\sigma \gg L_\Phi$. Here, gauge interactions will be efficient in correlating the phases of Φ and σ in the region around a given Φ-string and σ will be forced to wind with $N_\sigma = -N_\Phi$. However, when we consider a region of size greater than ξ_σ, the phase of σ will be random to the extent that σ may wind independent of any Φ-string. A volume of size ξ_σ then contains $O((\xi_\sigma/L_\Phi)^2)$ Nielsen-Olesen (i.e., $N_\Phi = -N_\sigma$) strings and $O(1)$ frustrated σ-string.

What values can one expect for L_Φ and ξ_σ? As mentioned in the introduction, the correlation length for a field at the time when it acquires a VEV is either $O(T^{-1})$ or $O(H^{-1})$ depending on the nature of the phase transition. On the other hand, the typical scale for strings in a network which has been evolving in the expanding Universe for at least a few Hubble times since formation is $O(H^{-1})$. For the situation under consideration, $L_\Phi = H^{-1}$. If $\xi_\sigma \sim T^{-1} \simeq (T/m_{pl})H^{-1}$, then $\xi_\sigma \ll L_\Phi$ and the Φ and σ strings will be independent. If $\xi_\sigma \sim H^{-1}$, then we $\xi_\sigma \sim L_\Phi$. This intermediate case is more difficult to analyze and numerical work is probably required in order to fully understand the string formation process. Still, since the winding numbers of Φ and σ are determined by conditions on the fields at some boundary outside of a correlation region, we can expect that in the absence of direct couplings between Φ and σ, the two fields will wind independently and the strings which form will be frustrated.

It is interesting to consider the case where Φ and σ acquire comparable VEVs and at roughly the same time in the history of the Universe. The correlation lengths for the two fields at the time of the phase transition will also be comparable. Again, we expect that the winding numbers of Φ and σ will be independent. With N_Φ, N_σ, q_Φ, and q_σ as fixed but arbitrary parameters, the coefficient of the $ln(R/r_o)$ term in the expression for the mass per unit length of the string will be $[\mu^2(N_\Phi - q_\Phi m)^2 + \eta^2(N_\sigma - q_\sigma m)^2]$ where $A'_\theta \to m/r$ far from the string. Minimizing this coefficient with respect to m, we find

$$m = \frac{\mu^2 q_\Phi N_\Phi \ + \ \eta^2 q_\sigma N_\sigma}{\mu^2 q_\Phi^2 \ + \ \eta^2 q_\sigma^2}. \tag{2.6}$$

The flux itself is not topological but depends upon the topological Higgs configurations. The two doublet configurations are, nonetheless, topologically stable in this case and yet do not produce an integer flux.

We now consider the case where the terms in V_{phase} are present and concentrate on the effects of these couplings. For $q_\Phi = -q_\sigma \equiv 1$, V_{phase} is given by

$$V_{phase}(\alpha, \beta) = a\cos(\gamma) \ + \ b\cos(2\gamma) \tag{2.7}$$

where we have neglected higher order terms. Though these terms can easily be included, they do not qualitatively change our results. First, consider the case where $a > 4b > 0$. σ can minimize its energy by choosing $\beta = \pi - \alpha$. Essentially, the effect of V_{phase} is to tilt the potential for σ. It is important to keep in mind that this is a local interaction, as distinct from the non-local gauge or derivative interactions discussed above. Clearly, if there is a Φ-string present with winding number N_Φ, then the groundstate configuration for σ has winding number $N_\sigma = -N_\Phi$ which corresponds to a Nielsen-Olesen string.

While V_{phase} determines the groundstate string configuration, the relaxation process that σ goes through to get to this state may be very complicated. In particular, if the phase potential for σ is only a small perturbation to the terms in the potential which determine the magnitude of σ, then we may expect that at the time when σ acquires a VEV, it *will not* necessarily wind with $N_\sigma = -N_\Phi$. Again, we can then have the situation where, in the region around an $N_\Phi = 1$ string, the σ field will initially have $N_\sigma = 0$. As V_{phase} turns on, σ will attempt to orient itself with $\beta = \pi - \alpha$. However, in order to do so over all of space, the magnitude of σ must pass through zero along some wall bounded by the string. The situation is similar to the case of the (1, 0) string previously discussed where there only gauge interactions were considered though the energetics in the present case is very different. In particular, if the time for the σ field to pass through its zero is very long, then a domain wall will be set up (Fig. 1). Though these walls are unstable to the formation of holes bounded by strings, they can be very long-lived.

For $4b > a > 0$ the phase potential for the σ field will be a double well potential with degenerate minima at $\beta = -\alpha \pm arccos\,(-a/4b)$. Using only local information one cannot determine a unique true vacuum configuration for σ. We expect that in different regions of the Universe, β will reside in different minima with domain walls forming between these regions. Inside the domain wall, $\beta = \pi - \alpha$ so that the energy density for the wall is $2b\left(1 + (a/4b)^2\right) - a = 2B\mu^2\eta^2\left(1 + (A/4B\mu\eta)^2\right) - A\mu\eta$ where $A \equiv A_1\mu^2 + A_2\eta^2$. The thickness of the wall is $\sim (B\mu^2)^{-1/2}$ so that its mass per unit area is given by $\sigma \equiv \left(2B\mu\eta^2\left(1 + (A/4\mu\eta)^2\right) - A\eta\right)/B^{1/2} \simeq B^{1/2}\mu\eta^2$.

Domain walls are in general disastrous for cosmology. Their formation and evolution has been discussed by a number of authors and here we simply list the highlights of their results. At the time of formation there are both closed surfaces and infinite walls. The subsequent evolution of this system is essentially governed by 1) surface tension, which acts to shrink closed surfaces and smooth out irregularities on infinite walls and 2) friction due to particle interactions which damp the motion of the walls. Balancing these two forces determines a critical scale ($\equiv R_c(t)$) such that closed surfaces smaller than R_c disappear in less than a Hubbles time and infinite walls become smooth on this scale. $R_c(t) \propto t^{3/2}$ and is equal to the horizon at a time $t_* \sim m_{pl}^2/\sigma$. At t_* one has $O(1)$ wall stretching across a horizon volume. For $t > t_*$, the energy density in the wall leads to distortions in the cosmic background radiation with $\delta\rho/\rho \sim 10^{60}(\sigma/m_{pl}^3)$ and this is incompatible with present limits on the microwave isotropy for $\sigma^{1/3} > 10^{-2}GeV$.

We now consider a situation where the ground state configuration for the σ field does not necessarily exist. Suppose that $2q_\Phi = -q_\sigma \equiv 2e$. Keeping only the leading terms in the phase potential, we have

$$V_{phase} = \frac{D}{2}\Phi\sigma^2 + h.c. = d\cos(\alpha + 2\beta) \tag{2.8}$$

where $d = D\mu\eta^2$. Again, the minimum for β is not uniquely determined. However, the topological structures in this case are somewhat different from the ones previously discussed. The true groundstate configuration will have winding numbers $N_\Phi = 2n$ and $N_\sigma = -n$ where n is an integer. But let us consider a Φ-string with $N_\Phi = 1$ and $\alpha = \theta$ (Fig. 2). Fig. (3) shows the vortex with a σ configuration which minimizes the potential everywhere locally (horizontal right directed short arrow means $\beta = 0$). We see that a defect occurs along the positive $x-$axis. That is, with $\beta = (\pi - \theta)/2$ we see that as $\theta = 2\pi \pm \epsilon$, $\beta = \pm\pi/2$ and since σ is single valued there must occur the defect. These defects are generic and arise by globally choosing β to fall into one of the double minima and finding an obstruction occurs somewhere, which defines the line defect. This defect is always present when $N_\Phi = odd$. If the Φ field has odd winding number, then σ must have global winding number $-odd/2$ which is incompatible with a single valued σ field, hence the domain wall forms. Since Φ forms a condensate at earlier times than σ, there is no way to ensure that $N_\Phi = even$.

It is not sufficient, however, that $N_\Phi = even$ to avoid the domain wall. The more subtle situation occurs in general for $N_\Phi = 2$ and incommensurate winding number configurations in σ. First, in Fig. (4) we show the true groundstate without domain wall with $N_\Phi = 2$ and $N_\sigma = -1$, which is obtained by choosing the phase of the σ field to lie in one of the two minima globally, e.g. $\beta = (\pi - 2\theta)/2$ globally.

However, this global information is not available to a random initial ensemble of σ fields and we must rely upon local information to construct the string state. In general we cannot decide locally into which minimum to place this phase. In Fig.(4) we show one choice of phase above the $x-$axis, $\beta = (\pi - 2\theta)/2$, and the other choice, $\beta = (3\pi - 2\theta)/2$, below it. In fact, one can see that the σ winding number in Fig. 4 is 0 (mod(2)) and not 1 (mod(2)) as required for the no domain-wall groundstate. This is accompanied by the formation of the domain and anti-domain walls emanating from the vortex corresponding to discontinuous jumps by $\pm\pi$ in the phase β.

Of course we can wrap the domain walls around the vortex so that they overlay one another and annihilate, their false vacuum energy being radiated away in scalar particles. The final configuration has winding number $N_\sigma = -1$ as in Fig. (3). The domain wall configuration is therefore unstable, but it is generally long lived.

Finally, we note that this model also contains infinite and closed surface domain walls. This is easily seen by considering a region of space in which Φ is constant in direction (i.e., no Φ-strings). The phase potential for σ then has degenerate minima at $(\pm\pi - \alpha)/2$. To summarize, the structures in this model are (1) $N_\Phi = odd$ strings which are always attached to domain walls; (2) $N_\Phi = even$ strings, which are sometimes accompanied by domain walls; and (3) infinite and (4) closed surface domain walls which are independent of any cosmic strings.

A system of domain walls bounded by strings [5] is a very complicated one. Such systems arise in a variety of models and in particular, when a discrete symmetry is embedded in a continuous group. As will be discussed below models with domain walls bounded by strings may be cosmologically 'safe' and therefore provide a way of having discrete symmetries without having a domain wall problem. We review the work on the evolution of these systems as it is relevant to the model under present consideration. In the model discussed here, the strings have a mass per unit length $\sim \mu^2$ so that the tension of a string with a curvature scale R ($\sim t$ in most cosmic string scenarios) is μ^2/R. The tension of the walls is σ. Therefore, for $R \sim t < \mu^2/\sigma$, the string evolution will be unaffected by the domain walls. For $t > \mu^2/\sigma$, the tension of the walls becomes dynamically important. The walls tend to pull the strings together in an attempt to minimize their surface area. The strings and walls intersect and the walls are 'cut' into pieces. These pieces lose energy by frictional losses, gravitational radiation, and particle radiation. Assuming that the intercommuting probability of *both* the strings and the walls is ~ 1, the system rapidly decays and disappears. Hence, there is no domain wall problem, but there are also no cosmic strings available for galaxy formation [5].

The above discussion is based on work which is very preliminary and moreover requires a number of unchecked assumptions. In particular, the question of the intercommuting probability for wall-wall and wall-string intersections has, to the best of our knowledge, not been carefully analyzed. If this probability is ~ 0 then the domain walls will survive until late times and this is incompatible with present observations. On the other hand, it may be possible for the structures to survive through the time when galaxies and clusters are entering the horizon so that these objects could be important for the formation of large scale structure. We also note that if there is a period of supercooling (mini-inflation) between the time when Φ and σ acquire VEVs, then the Φ strings will be pushed well outside the horizon. One then has the situation where there are σ domain walls with no Φ-strings and hence a domain wall problem. The issues raised here are very model dependent. Furthermore, a complete understanding of some of these problems will most likely require numerical simulations. Some of these problems will be the subject of future investigations.

Finally, we consider a model with two $U(1)$ symmetries and three scalar fields where the charge assignments are given in Table I. The leading terms in the phase potential are $V_{phase} = A\Phi\sigma\lambda \ + \ h.c.$. These terms imply that for any groundstate, Nieslen-Olesen string the sum of the winding numbers of the three fields must vanish:

$$N_{\Phi} \ + \ N_{\sigma} \ + \ N_{\lambda} \ = \ 0. \tag{2.9}$$

In particular, the three groundstate strings [i.e., ones with winding numbers $(\pm 1, 0)$] are $N_{\Phi} = -N_{\sigma} = 1$, $N_{\Phi} = -N_{\lambda} = 1$, and $N_{\sigma} = -N_{\lambda} = 1$. In each case, one of the fields will be non-zero in the core of the string.

The model described above is of particular importance for understanding how the fermionic superconducting cosmic string might arise in various grand unified models [6,7]. Fermions which get a mass through Yukawa couplings to the scalar fields which make up a Nielsen-Olesen string are trapped as massless particles in the core of the string [10]. These particles move along the string at the speed of light. If they also carry electromagnetic charge, then they can potentially carry enormous currents [6] which may have important implications for galaxy formation [11] and for the production of ultra-high energy cosmic rays [12]. However, if the fermions also couple to a scalar field which is non-zero in the core of the string then the fermions will no longer be massless in the core and the superconductivity will be spoiled [9]. In effect, the mass terms act as a capacitance to the current and, in most physically realistic scenarios, this prevents the build up of large currents.

III. Realistic Models

The examples discussed in the previous section are easily realized in standard Grand Unified Theories (GUTs). Let us first consider cosmic strings within the context of an $O(10)$ GUT [6,7]. $O(10)$ is broken to $SU(5) \times U(1)'$ via the vev of a Higgs field in the adjoint, i.e. a 45. Limits on the mass of the extra Z boson associated with $U(1)'$ require that $U(1)'$ be broken at a scale $\geq O(300GeV)$ [13]. The breaking of $U(1)'$ must therefore be accomplished by a standard model singlet. The natural choices are the $1^{-5/2}$ component of the 16 or the 1^{5} component of the 126, denoted by N and Δ respectively, where the superscripts refer to the $U(1)'$ charges of the fields. The conventional see-saw mechanism for obtaining ultralight neutrino masses [14] favors a large vev for Δ ($\langle \Delta \rangle \geq 10^{7}GeV$ if one wishes $m_{\nu_\tau} \leq 100eV$). Breaking of $U(1)'$ via $\langle \Delta \rangle \neq 0$ will lead to the formation of stable strings [6]. We assume that N obtains a vev, but at a much later time so that $\langle N \rangle \ll \langle \Delta \rangle$. Now suppose the Higgs potential contains the trilinear coupling $A\Delta NN \subset A \cdot 126 \cdot 16 \cdot 16$ where A is an unspecified mass parameter. The situation identical to the example discussed in Section II where $V_{phase} = \Phi\sigma^2$. For a Δ string with odd winding number, local correlations of N and Δ resulting from the above couplings will force N to form a domain wall attached to the string.

We now illustrate how, within a realistic GUT, fermionic superconductivity in cosmic strings can be spoiled due to the presence of non-zero vev's in the string core. The model considered is a conventional E_6 based GUT [6]. We will also comment briefly on cosmic strings in the context of superstring-type E_6 models [7].

E_6 can be broken to $O(10) \times U(1)'$ via the vev of a Higgs in the adjoint, i.e.

a 78. The remaining Higgs and all of the fermions will be contained in $27's$ of E_6. In Table II, we list the neutral Higgs and the charged fermions with their transformation properties under $SU(5) \times U(1)'' \times U(1)'$. The Yukawa potential for the entries in Table II consists of the following terms:

$$27^3 \supset N^c e E^c + N^c h d^c + n^c h h^c + n^c E E^c + \nu e^c E + \nu d h^c + \nu_e d d^c + \nu_E e e^c + N_E^c u u^c + n \nu_E N_E + N \nu N_E. \tag{3.1}$$

The superscript 'c' denotes charge conjugation. A vev for n breaks $U(1)'$ and leads to strings which are essentially stable, with lifetimes $\geq O\left(10^{10^4}\right)$ if $\langle n \rangle \leq .1\mu M_{GUT}$ [6]. We assume that n is the first scalar to obtain a vev. The vev of N is constrained to be $\geq O(300GeV)$ [13] while those of ν_E, N_E, and ν will have to be $\leq O(M_W)$.

Assume that the following scalar trilinears are present in V_{phase}:

$$V_{phase} \supset A n \nu_E N_E + B N \nu N_E. \tag{3.2}$$

Local phase correlations then lead to the following relations among the winding numbers of the Higgs fields:

$$N_n + N_{\nu_E} + N_{N_E} = 0 \qquad N_N + N_\nu + N_{N_E} = 0. \tag{3.3}$$

It is readily checked that these relations are sufficient to insure formation of local rather than global string configurations. Consider the following minimal ($|N_X| \leq 1$) string configuration which satisfies

$$N_n = 1, \quad N_{\nu_E} = -1, \quad N_N = N_\nu = N_{N_E} = 0 \tag{3.4}$$

so that N, ν, N_E are, in general, non-vanishing in the core. EE^c, hh^c, dd^c, and ee^c which obtain their masses through Yukawa couplings with either n or ν_E are trapped in the string. However, due to their couplings to N, ν, and N_E which are non-zero in the core, these fermions will be massive in the core and therefore, the superconductivity will be spoiled[9]. More generally, for minimal string configurations, only a subset of the would-be zero modes will remain massless in the core. Moreover, from Eqn(3.3), it is clear that to obtain the simple superconducting cosmic string scenario, where all of the zero modes do indeed remain massless in the core, one must require non-minimal (i.e., higher winding number) string configurations.

Next we show that realistic supersymmetric or superstring-type E_6 based models with intermediate mass scales generally will contain global strings. In what follows we consider, for simplicity, the Higgs sector of Table II. The argument generalizes to the vector-like Higgs sector of superstring-type models, where intermediate mass scales for n, N, and their mirrors are easily generated along D-flat directions. A large vev for n or N ($\langle n\rangle$ or $\langle N\rangle \gg TeV$) necessitates the absence of superfield Yukawa couplings $\hat{n}\hat{N}_E\hat{\nu}_E$ and $\hat{N}\hat{\nu}\hat{N}_E$, respectively ($\hat{}$ denotes a superfield). Otherwise, the pair of scalars N_E and ν_E or N_E and ν which acquire mass contributions from F^2 terms of $O(\langle n\rangle^2)$ and $O(\langle N\rangle^2)$ respectively, will be too massive to obtain non-negligible vev's. However such vev's are required both for $SU(2)_L$ breaking and fermion mass generation. The absence of either one of the above couplings from the superpotential would in turn lead to an absence of the corresponding scalar trilinear in V_{phase} (see Eqn(3.2) since the latter is induced by the former through SUSY breaking (with A, and $B \leq m_o$, the SUSY breaking scale). The absence of either one of the couplings for V_{phase} means that local phase correlations between the Higgs of Table II will not be sufficient to insure that only local strings form. Given that the gauge interactions are, in general, not able to insure that the different fields wind as they must in order to have local Nielsen-Olesen-type strings, we expect energetically 'global' strings. It has been demonstrated that fermionic superconducting cosmic strings can exist in superstring-type E_6 models [7]. However, 'superconductivity' will be very different for these 'global' strings and it is not clear whether they can support large currents [15].

Finally, we note that in E_6 based superstring-type models, the non-renormalizable couplings $\lambda(nn')^{m+2}/m_{pl}^{2m+1}$ or $\lambda(NN')^{m+2}/m_{pl}^{2m+1}$ (where prime refers to the corresponding higgs in the $\bar{27}$, λ is a dimensionless coupling constant, and $m \geq 0$) are often introduced to lift flat directions in the scalar potential, leading to large intermediate mass scale vev's $\geq O(10^{10}GeV)$ for n^c and n' or N^c and N'. The above couplings are expected to induce, via supergravity breaking effects, the scalar couplings $\lambda A(nn')^{m+2}/m_{pl}^{2m+1}$ or $\lambda A(NN')^{m+2}/m_{pl}^{2m+1}$ where the magnitude of A is currently not known but is expected to be $\leq TeV$, the observable SUSY breaking scale. These scalar couplings, which are a generalization of the $\sigma^2\Phi^2$ terms considered in Section II lead to domain walls which are independent of the cosmic strings. These domain walls will be cosmologically unacceptable. As an example, for $m = 0$ the VEVs generated will be $O(10^{10}GeV)$ so that the resulting deviations from the microwave isotropy will exceed current bounds if $\lambda A > 10^{-63} GeV$. Therefore if SUSY breaking does lead to $A \neq 0$ then the above considerations provide a constraint for model building.

IV. Conclusion

Cosmic strings without domain walls are favored in astrophysics. However, the topological structures which form in realistic models in which there are many $U(1)'$-charged scalars are, in general, more complicated. While local, Nielsen-Olesen type strings are often acceptable solutions in these models, these strings require special conditions on the winding numbers of the different fields. Winding numbers, however, are determined by conditions on the fields in remote regions of space and are therefore random in origin. Rather than a system of local strings, one is more likely to have a system of global-like strings and perhaps strings with associated domain walls. The type of structures which form and their subsequent evolution depend quite crucially on the particular particle physics model one is studying.

We have by no means exhausted the problems presented here and we hope that this work spurs further investigations along these lines.

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Figure Captions

1. Figure 1: Field configuration where the heavy $\Phi = \mu e^{i\alpha}$ field (long arrows) has winding number 1 and the light $\sigma = \eta e^{i\beta}$ field (short arrows) has winding number 0 but feels the effect of a potential $V_{phase} \propto \cos(\alpha + \beta)$ which has a minimum at $\beta = \pi - \alpha$. [A right directed arrow denotes α or $\beta = 0$.] The potential is minimized everywhere except along the $\theta = 0$ axis where $\alpha + \beta = 0$ and a domain wall (albeit unstable one) appears.

2. Figure 2: $N_\Phi = 1$ string.

3. Figure 3: $N_\Phi = 1$ string with σ forming a domain wall along $\theta = 0$ (positive x) axis.

4. Figure 4: $N_\Phi = 2,\ N_\sigma = -1$ string.

5. Figure 5: $N_\Phi = 2,\ N_\sigma = 0(mod(2))$ string with σ forming a domain wall.

Table I

	Q	R
Φ	2	0
σ	-1	1
λ	-1	-1

Table I: Charge assignments for $U(1)' \times U(1)''$ model with three scalar fields.

Table II

scalars	$SU(5) \times U(1)'' \times U(1)'$	fermions	$SU(5) \times U(1)'' \times U(1)'$
n^c	$(1,\ 0,\ 4)$	$E,\ E^c$	$(\bar{5},\ 2,\ -2,),\ (5,\ -2,\ -2)$
N	$(1,\ -5,\ 1)$	$d,\ d^c$	$(10,\ -1,\ 1),\ (\bar{5},\ 3,\ 1,)$
ν_E	$(\bar{5},\ -2,\ -2)$	$h,\ h^c$	$(5,\ 2,\ -2),\ (\bar{5},\ -2,\ -2)$
N_E	$(5,\ 2,\ -2)$	$u,\ u^c$	$(10,\ -1,\ 1),\ (10,\ -1,\ 1))$
ν	$(\bar{5},\ 3,\ 1)$	$e,\ e^c$	$(\bar{5},\ 3,\ 1),\ (10,\ -1,\ 1)$

Table II: Charge assignments for neutral scalars and charged fermions in the **27** of E_6.

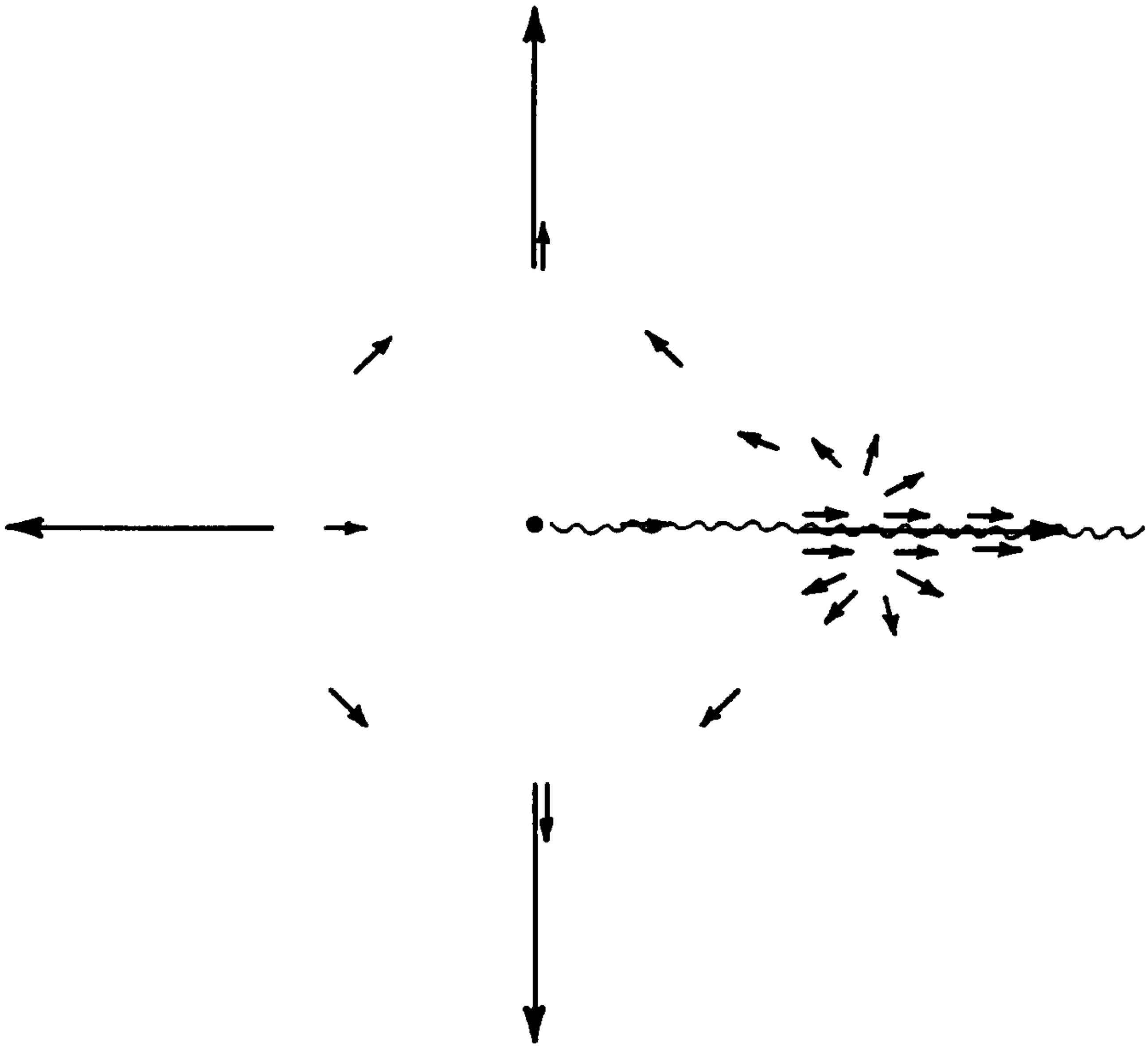

Figure 1

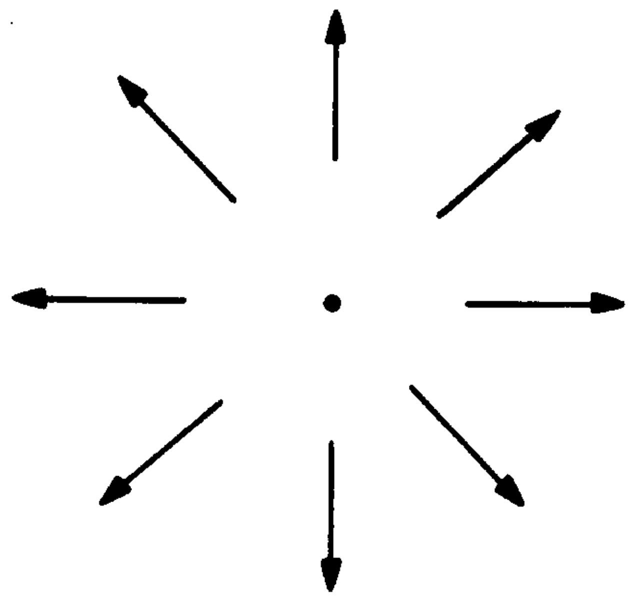

Figure 2

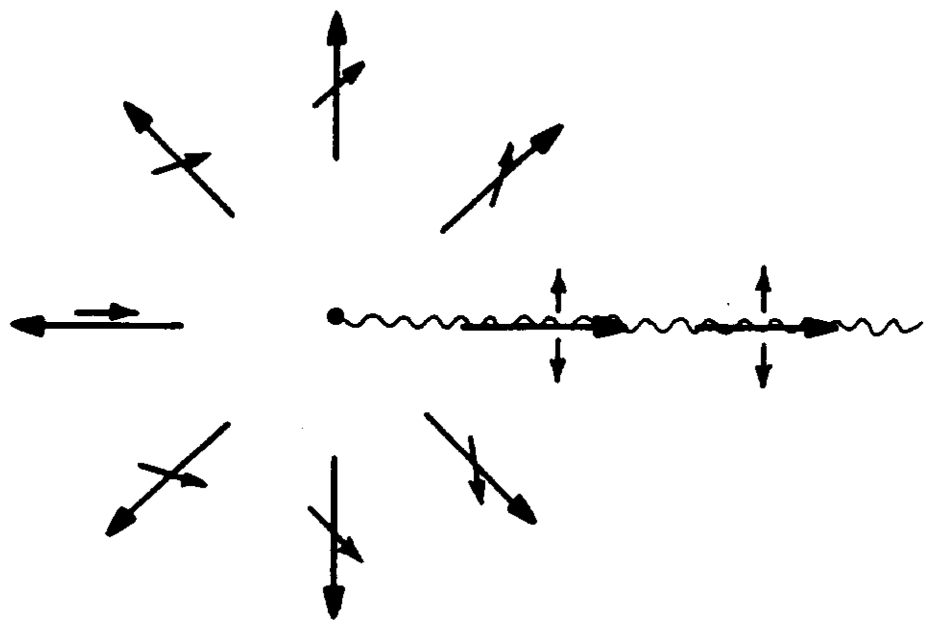

Figure 3

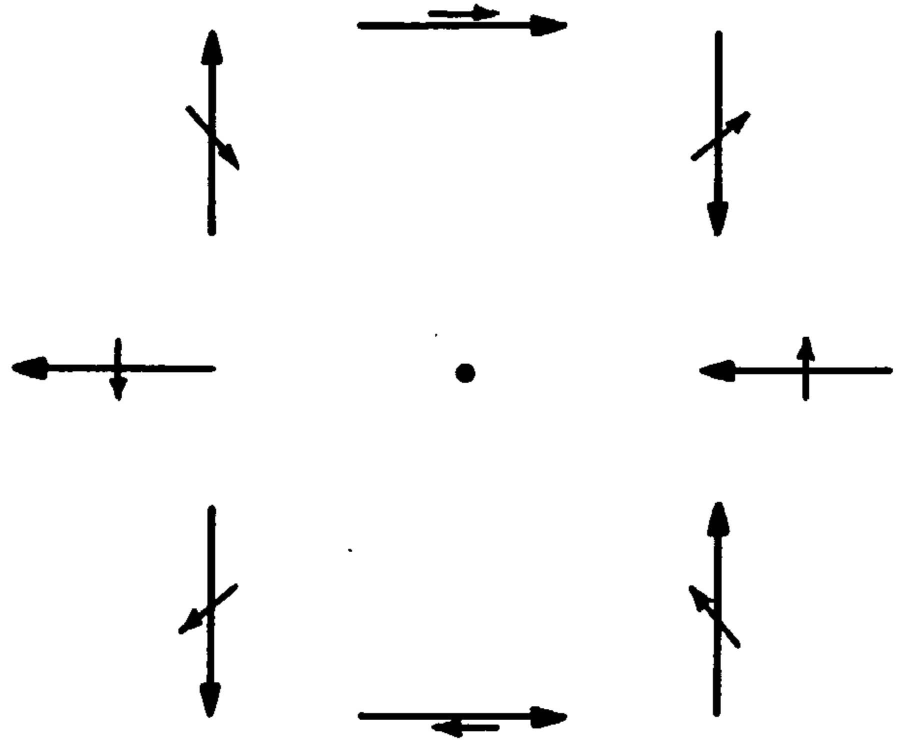

Figure 4

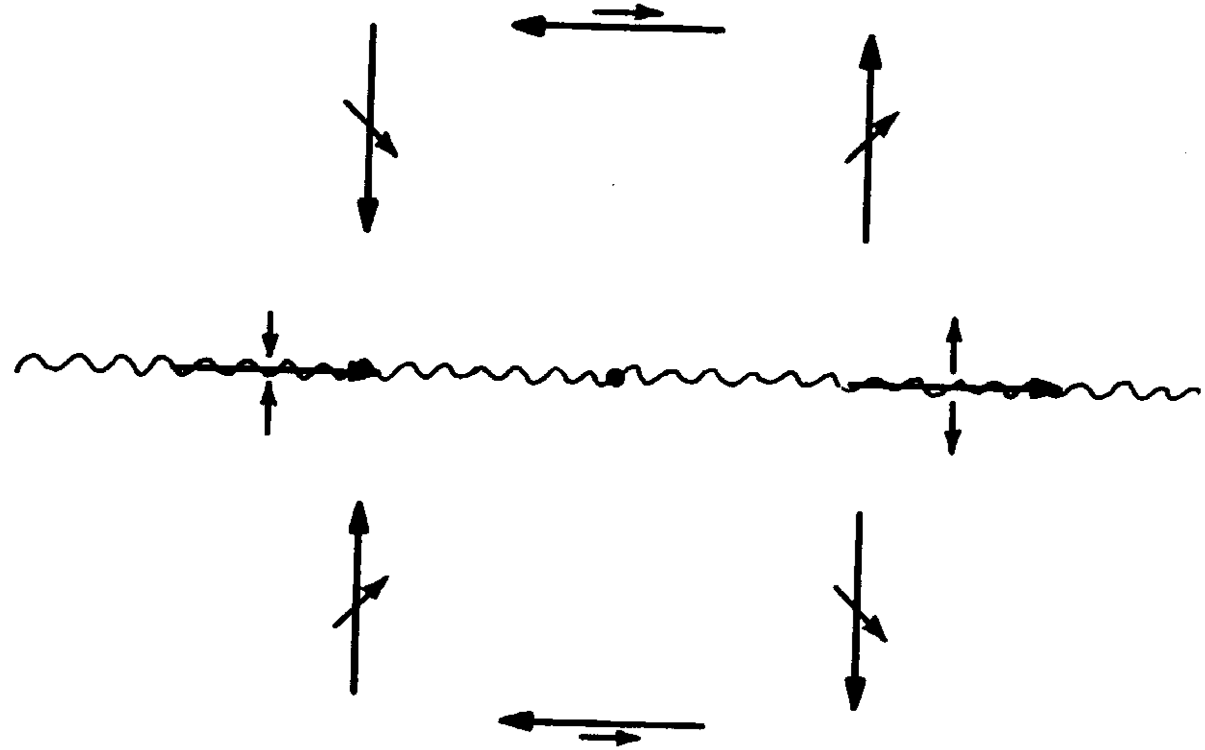

Figure 5